

# Worker and Firm Productivity Effects: A New Approach to Human Capital Spillovers Within Firms

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## ***Abstract***

This study investigates human capital externalities within firms by comparing the determinants of productivity and wages at firm and worker level. The firm-level analysis provides improved estimates based on an extended set of observables including the intensity of firm-provided training, while the analysis at worker level takes a new turn by generating a proxy for unobserved worker productivity. We also examine the magnitude of spillovers based on a new approach that compares firm and worker productivity estimates. Our results point to the presence of a multiplier effect in the provision of schooling and training, as benefits from individual human capital acquisition are evidently spread over co-workers. We found that in the case of schooling 75% of the marginal benefit is captured by firms, 21.5% by co-workers, and 3.5% by the worker. These new estimates have the notable advantage of being extracted from productivity estimates rather than from observed wages.

Keywords: Inter-firm spillovers, worker productivity, wages, human capital, *LEED*.  
JEL Codes: C23, D24, J31

## **1. Introduction**

The majority of studies that investigate human capital investment, in particular, the literature on schooling effects, rely on private benefits. However, it is not less important the understanding of whether, say, an individual with a higher schooling level increases his/her productivity or also the productivity of his/her peer workers. Since most tasks are performed in group, it is expected that by sharing information through formal and informal interactions workers with increased skills generate human capital spillovers. In this sense, the acquisition of human capital ought to be profitable for the worker who invests in human capital, his/her co-workers, and the firm.

Some of the studies investigating human capital spillovers have been interested in evaluating externalities across individuals in a given geographic area. For example, Moretti (2004) found that an increase in the supply of cities' college graduates has a significant wage effect, especially on less qualified workers. But presumably a more direct, within-firm effect should be expected as human capital externalities arise mainly through worker interaction in the workplace (Martins and Jin, 2010). There is indeed an increasing literature that examines the within-firm effect of education (e.g., Battu, Belfield and Sloane, 2003, Martins and Jin 2010, Raymond and Roy, 2011). For example, Battu, Belfield and Sloane (2003) found that the social return to education (proxied by the average workplace education) is strongly positive. An important shortcoming of these studies, however, is that they rely on the impact of schooling on wages, rather than on productivity. Since the external effects of human capital investment are expected to be larger on firm productivity than on wages, the spillover effect is bound to be underestimated.

The main goal of this paper is to investigate human capital spillover effects based on an original approach that uses the estimated differences across productivity effects at firm and worker level. To this end, we derive an unique analytical framework for the determination of worker-level productivity – one that allows us to connect human capital variables with individual productivity, while, at the same time controlling for the unobserved worker and firm effects – to then estimate the magnitude of the spillover effects. In the process we extend the examination of human capital externalities to training and other observables. Finally, we

provide an estimate of the social benefits arising from worker investments and how these benefits are shared by workers, co-workers and firms.

Our modeling at firm level follows the pioneering work of Hellerstein, Neumark and Troske (1999), that is, we set a firm-level Cobb-Douglas production function in which labour is the product of hours worked multiplied by ‘quality’, where quality is a function of observables. Then, based on a standard Mincerian earnings equation, we derive the corresponding firm-level wage equation. Since both productivity and wages are a function of a common set of regressors, we jointly estimate the two equations in order to infer the extent to which the productivity gains from schooling and training, inter al., are shared by firms and workers.

In a second stage, we estimate a proxy for individual productivity using the wage distribution within firms. This means that we do not use the strong assumption of a strict correspondence between individual productivity and earnings. We do not require the same mean and standard deviation across the wage and productivity distributions either. In turn, we use the assumption that the two distributions have approximately the same coefficient of variation, a condition that is easily satisfied in our data. Then, by aggregating the generated individual productivity across workers, we show that the corresponding firm productivity level is highly correlated with the sample productivity level.

Finally, by using the estimates from the productivity equation, both at worker and firm level, we derive the spillover effect. In particular, our modeling strategy relies on the assumption that in the absence of any spillover effect, and after controlling for observables, the marginal effect of worker  $i$ 's investment in schooling on firm productivity should be exactly equal to the worker's marginal effect divided by the number of workers in the firm. In this framework, any difference between the two quantities is defined as a spillover.

For illustration purposes, we use a 2-year long LEED panel comprising 288,129 workers and 1,174 firms from the Portuguese manufacturing sector. In our data, which is extracted from two data sources (i.e. Balanço Social and Quadros de Pessoal), we are able to follow workers and firms longitudinally. We also observe individual – worker and firm – characteristics, including detailed information on firm-provided training extracted from

Balanço Social. The latter is a relevant aspect as studies solely based on Quadros de Pessoal cannot take into account the workplace training variable.<sup>1</sup>

This paper is organized as follows. In the next section we present the modeling of the determinants of productivity and wages, as well as the full derivation of our selected proxy for worker productivity and the corresponding estimate of the spillover effect. Section 3 describes the longitudinal LEED dataset used in our illustration, while Section 4 discusses the generated results. The main conclusions are drawn in Section 5.

## 2. Modelling

### 2.1 Firm productivity

We start by considering a Cobb-Douglas production function given by

$$Y_{jt} = AL_{jt}^{\alpha} K_{jt}^{\beta} e^{\left(\sum_{p=1}^P \eta_p Z_{p,jt} + \pi \psi_j + \varepsilon_{jt}\right)}, \quad (1.1)$$

where  $Y_{jt}$  denotes the value added of firm  $j$  in period (year)  $t$ .  $A$  is an efficiency parameter,  $K_{jt}$  is the stock of capital,  $Z_{p,jt}$  is a set of  $P$  firm observed characteristics (e.g. location),  $\psi_j$  is the (time-invariant) unobserved heterogeneity of firm  $j$ , and  $\varepsilon_{jt}$  denotes the *i.i.d.* error term.<sup>2</sup>  $L_{jt}$  is the labour input, given by  $L_{jt} = h_{jt} * V_{jt}$ , where  $h$  is hours of work per employee, and  $V$  is a labour composite comprising various human capital and demographic characteristics.

A possible specification for the (log) hourly productivity of labour,  $\ln y$ , is given by:<sup>3</sup>

$$\begin{aligned} \ln y = & \ln A + \alpha(\gamma_G - 1)G + \alpha(\gamma_E - 1)E + \alpha(\gamma_T - 1)T + \alpha(\gamma_S - 1)S + \alpha(\gamma_O - 1)O + \\ & + \alpha(\gamma_{Q_1} - 1)Q_1 + \alpha(\gamma_{Q_2} - 1)Q_2 + \alpha(\gamma_{Q_3} - 1)Q_3 + \alpha(\gamma_{Q_4} - 1)Q_4 + \alpha(\gamma_{Q_5} - 1)Q_5 + \\ & + \beta \ln k + \sum_{p=1}^P \eta_p Z_p + \pi \psi + \varepsilon, \end{aligned} \quad (1.2)$$

with  $G$ ,  $E$ ,  $T$ ,  $S$ , and  $O$  denoting the corresponding proportion of male workers in the firm,

<sup>1</sup> Lopes and Teixeira (2013) provide a detailed analysis of workplace training using *Balanço Social*.

<sup>2</sup>  $\psi_j$  gives the worker average unobserved ability in firm  $j$ , plus an unknown firm specific effect (or  $\bar{\alpha}_j + \phi_j = \psi_j$ ). See Appendix A.

<sup>3</sup> We omit subscripts  $j$  and  $t$  hereafter. The full derivation of model (1.2), which is inspired in Hellerstein, Newmark, and Troske (1999), is available upon request.

workers with at least a high-school degree, training participants, workers with at least 10 years of service and workers between 25 and 44 years old. We also introduce several job occupation categories: top managers and professionals ( $Q_1$ ), other managers and professionals ( $Q_2$ ), foremen and supervisors ( $Q_3$ ), highly skilled and skilled personnel ( $Q_4$ ), and semiskilled personnel ( $Q_5$ ). The variable  $\gamma_G$ , for example, is the marginal contribution of a male worker relatively to a female worker.  $\ln k$  denotes the (log) capital intensity.

Now, in sake of simplicity, equation (1.2) will be described in matrix notation. Thus, for  $J$  firms observed over  $T$  periods, we have:

$$Y = X\tilde{\gamma} + K\beta + Z\eta + F\psi\pi + \varepsilon, \quad (1.3)$$

where  $Y$  and  $K$  are two vectors of  $JT$  elements (in logs). The matrix  $Z$  contains the observed firm characteristics.<sup>4</sup>  $F$  is a  $(JT \times J)$  matrix of dummies flagging the  $J$  firms, while  $X$  contains 10 (firm-level) worker characteristics,  $G, E, T, O, S, Q_1, Q_2, Q_3, Q_4, Q_5$ , so that  $\tilde{\gamma}$  is

$$\text{given by } \tilde{\gamma} = \begin{bmatrix} \alpha(\gamma_G - 1) \\ \dots \\ \alpha(\gamma_{Q_5} - 1) \end{bmatrix}_{10 \times 1}.$$

## 2.2 Earnings equation

At the worker level, and following Abowd, Kramarz and Margolis (1999) model, the augmented Mincerian wage equation can be given by:

$$W = X'\lambda^i + K'\beta_w^i + Z'\eta_w^i + C\theta + F'\psi + \xi, \quad (2.1)$$

where  $W$  is an  $N^s T$ -element column vector denoting the log hourly earnings of each worker,  $\ln w_{it}$ , with  $i=1, 2, \dots, N^s$ . The sample contains  $N^s$  workers who are observed over  $T$  periods.  $X'$  is a  $N^s T \times 10$  matrix containing all the dummy variables flagging worker-level characteristics.<sup>5</sup>  $K'$  and  $Z'$  denote capital and other firm observable characteristics,

<sup>4</sup> To simplify the notation, we include  $\ln A$  in  $\eta$  and add a column of 1s to  $Z$ .

<sup>5</sup> The first row vector in  $X'$  is given by:

$(G'_{i1} \quad E'_{i1} \quad T'_{i1} \quad O'_{i1} \quad S'_{i1} \quad Q'_{1,i1} \quad Q'_{2,i1} \quad Q'_{3,i1} \quad Q'_{4,i1} \quad Q'_{5,i1})$ , now defined at worker level, with the first subscript flagging the worker and the second the year in which the individual is observed.

respectively, while  $\psi$  denotes firm unobserved effects.<sup>6</sup>  $C$  denotes a  $N^S T \times N^S$  matrix of dummies flagging the worker over the sample period  $T$ ;  $\theta$  denotes the (time-invariant) unobserved worker ability in deviation form from the worker average unobserved ability in firm (see Appendix A).

Summing up across workers in firm  $j$  in period  $t$ , and dividing by  $N_{jt}$  (i.e. the number of workers in firm  $j$  in period  $t$ ), we have, in matrix notation, the general model for the average wage (in logs):<sup>7</sup>

$$\bar{W} = X\lambda + K\beta_w + Z\eta_w + F\psi + e, \quad (2.2)$$

where  $\bar{W}$  is a column vector containing  $JT$  elements given by  $\frac{1}{N_{jt}} \sum_{i=1}^{N_{jt}} \text{Ln } w_{it}$ , the average

log hourly earnings in firm  $j$  at period  $t$ . Now, as we want to directly compare the determinants of firm productivity with the determinants of the firm wage – or models (1.3) and (2.2), respectively – we follow Hellerstein, Neumark and Troske (1999) and replace, in

equation (2.2),  $\frac{1}{N_{jt}} \sum_{i=1}^{N_{jt}} \text{Ln } w_{it}$  by  $\text{Ln } w_{jt} = \text{Ln } \frac{1}{N_{jt}} \sum_{i=1}^{N_{jt}} w_{it}$ . Clearly, given that the log

function is concave, we have, by Jensen's inequality,  $\text{Ln } \frac{1}{N_{jt}} \sum_{i=1}^{N_{jt}} w_{it} > \frac{1}{N_{jt}} \sum_{i=1}^{N_{jt}} \text{Ln } w_{it}$ , and

therefore  $\lambda$  is expected to be different from  $\lambda^i$ .

Once obtained the firm fixed effects,  $\hat{\psi}_j$  (see Appendix A), they can be inserted into models (1.3) and (2.2) to obtain, respectively:<sup>8</sup>

$$Y = X\tilde{\gamma} + K\beta + Z\eta + \tilde{\psi}\pi + \varepsilon, \quad (2.3)$$

and

$$W = X\lambda + K\beta_w + Z\eta_w + \tilde{\psi} + e. \quad (2.4)$$

$E'_{it}$ , for example, is equal to 1 if worker  $i$  has at least a high-school degree, 0 otherwise. And similarly for all the other worker-level covariates. See Section 3 for a full description of the variables and data sources.

<sup>6</sup>  $K'$ ,  $Z'$  and  $F'$  are distinct from  $K$ ,  $Z$  and  $F$  as the number of rows in equation (1.3) is  $JT$  while in (2.1) is  $N^S T$ .

<sup>7</sup> By summing up across workers in the same firm, the first element of  $X'$ , for example, will be given by

$\sum_{i=1}^{N_{jt}} G'_i = N^M_{jt}$  - the number of male workers in the firm, which, after dividing by the number of workers in firm  $j$

in period  $t$ , yields  $G_p (= \frac{N^M_{jt}}{N_{jt}})$ , or the first element of  $X$ . The same for the remaining worker's characteristics.

Accordingly, we can interpret  $\lambda_g$  as the gender wage gap. Note that, by definition, we have  $\sum \theta_i = 0$  (see Appendix A).

<sup>8</sup> To simplify our modulation, we set  $\tilde{\psi} = F\hat{\psi} = \begin{bmatrix} \hat{\psi}_1 & \hat{\psi}_1 & \hat{\psi}_2 & (\dots) & \hat{\psi}_j \end{bmatrix}^T_{1 \times JT}$ .

Now, as both the productivity and wages at firm level are a function of the same set of regressors, we are in a position to examine whether a given covariate has a bigger impact on wages than on productivity, or the other way around.

Note also that by construction,  $\hat{\psi}_j$  contains the average (unobserved) worker attributes. This means that in our firm-level equations we do control for the possible correlation between unobserved ability and the observed characteristics of workers at the firm level. This is a non-trivial aspect of our modelling strategy.

The relative impact of human capital and demographic characteristics can also be tested by running, again at firm level, the wage-productivity ratio,  $D_{jt}$ , on the common set of regressors. In this case, using  $D_{jt} = \frac{W_{jt}}{Y_{jt}} = \frac{w_{jt}}{y_{jt}}$ , we have  $\text{Ln } D_{jt} = \text{Ln } w_{jt} - \text{Ln } y_{jt}$ . Then, using equations (2.3) and (2.4),  $\text{Ln } D_{jt}$  can be estimated by (in matrix notation):<sup>9</sup>

$$D = X(\lambda - \tilde{\gamma}) + K(\beta_w - \beta) + Z(\eta_w - \eta) + \tilde{\psi}(1 - \pi) + v. \quad (2.5)$$

For example, the tenure term in  $(\lambda - \tilde{\gamma})$ , given by  $\lambda_s - \alpha(\gamma_s - 1)$ , is expected to be positive, as tenure is assumed to have a bigger impact on wages than on productivity. By the same token, if  $\psi_j$  flags non-competitive high-wage firms, then it will be expected to be associated with a larger w/y ratio.

### 2.3 An estimate for worker productivity

In this section, we extend the investigation on the determinants of productivity and wages by estimating the wage and productivity equations at worker level. We assume, in particular, that the (log) productivity of worker  $i$  in period  $t$ ,  $\text{Ln } y_{it}$ , is a function of the same set of covariates as specified in the worker level wage equation (2.1), which means that  $\text{Ln } y_{it}$  depends on both observed and unobserved worker and firm characteristics, that is:<sup>10</sup>

$$Y' = X' \tilde{\gamma}^i + K' \beta^i + Z' \eta^i + \tilde{\theta} \kappa + \tilde{\psi}' \tau + \mu, \quad (3.1)$$

<sup>9</sup> The first element of the vector column  $D$  is given by  $\text{Ln } D_{11}$ .

<sup>10</sup> Equation (A.8) in Appendix A is used to estimate the unobserved worker effects,  $\hat{\theta}$ . To simplify the notation, we set  $\tilde{\theta} = C \hat{\theta}$  and  $\tilde{\psi}' = F' \hat{\psi}$ .

where  $Y'$  is an  $N^S T$ -element column vector of (log) individual productivities,  $\text{Ln } y_{it}$ .

Since  $y_{it}$  (or  $\text{Ln } y_{it}$ ) is unobservable, we next show that it is possible to find an estimate of  $\text{Ln } y_{it}$  (call it  $\text{Ln } y_{it}^*$ ) so that one can estimate (3.1) and compare with (2.3) in order to investigate the presence of human capital externalities within firms.<sup>11</sup> Our approach is to use the wage distribution within firms to infer about individual (worker) productivity.

We start by noting that (a) we do observe  $\text{Ln } y_{jt}$ , where  $y_{jt} = \frac{1}{N_{jt}} \sum_{i=1}^{N_{jt}} y_{it}$  is the productivity level of firm  $j$  in period  $t$ , and (b)  $\text{Ln} \left( \frac{1}{N_{jt}} \sum_{i=1}^{N_{jt}} y_{it} \right) > \frac{1}{N_{jt}} \sum_{i=1}^{N_{jt}} \text{Ln } y_{it}$  or

$\text{Ln } y_{jt} - \frac{1}{N_{jt}} \sum_{i=1}^{N_{jt}} \text{Ln } y_{it} > 0$  (by Jensen's inequality). By Jensen's inequality we also have (c)

$$\text{Ln } w_{jt} - \frac{1}{N_{jt}} \sum_{i=1}^{N_{jt}} \text{Ln } w_{it} > 0.$$

Next, we assume the equality between (b) and (c):

$$\text{Ln } w_{jt} - \frac{1}{N_{jt}} \sum_{i=1}^{N_{jt}} \text{Ln } w_{it} = \text{Ln } y_{jt} - \frac{1}{N_{jt}} \sum_{i=1}^{N_{jt}} \text{Ln } y_{it}. \quad (3.2)$$

We note that condition (3.2) is very convenient in the sense that it does not imply a strict correspondence between individual (worker) productivity and earnings. Nor does it imply that the two distributions have the same mean and standard deviation either. As shown in Appendix B, what is required is that the two distributions have approximately the same coefficient of variation, a requirement easily satisfied in our data. Finally, our approach has the interesting property of replicating the sample firm productivity level once we aggregate the generated individual productivity across all workers in a given firm, as we will see in Section 4.2 below.

By further manipulating (3.2) we have:

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<sup>11</sup> Additionally, by comparing (3.1) and (2.1) we extend Hellerstein, Neumark and Troske (1999) model to the worker level and control for differences in unobserved worker effects.



$$\begin{aligned}
& \frac{1}{N_{jt}} \sum_{i=1}^{N_{jt}} \text{Ln } w_{it} - \frac{1}{N_{jt}} \sum_{i=1}^{N_{jt}} \text{Ln } y_{it} = \text{Ln } w_{jt} - \text{Ln } y_{jt} \Leftrightarrow \\
& \Leftrightarrow \frac{1}{N_{jt}} \sum_{i=1}^{N_{jt}} (\text{Ln } w_{it} - \text{Ln } y_{it}) = \text{Ln } w_{jt} - \text{Ln } y_{jt} \Leftrightarrow \\
& \Leftrightarrow \frac{1}{N_{jt}} \sum_{i=1}^{N_{jt}} \text{Ln } D_{it} = \text{Ln } D_{jt}, \tag{3.3}
\end{aligned}$$

with  $D_{it} = \frac{w_{it}}{y_{it}}$ .

Now, given  $\text{Ln } D_{it} = \text{Ln } w_{it} - \text{Ln } y_{it}$  and using (2.1) and (3.1), we can obtain, in matrix notation, the log wage-productivity gap at worker level:

$$D' = X'(\lambda^i - \tilde{\gamma}^i) + K'(\beta_w^i - \beta^i) + Z'(\eta_w^i - \eta^i) + \tilde{\theta}(1 - \kappa) + \tilde{\psi}'(1 - \tau) + (\xi - \mu), \tag{3.4}$$

which in turn, averaging over all individuals in firm  $j$ , yields (see footnote 7 above):

$$D^* = X(\lambda^i - \tilde{\gamma}^i) + K(\beta_w^i - \beta^i) + Z(\eta_w^i - \eta^i) + \tilde{\psi}(1 - \tau) + (\xi - \mu). \tag{3.5}$$

( $\frac{1}{N_{jt}} \sum_{i=1}^{N_{jt}} \text{Ln } D_{it}$  is the general element of  $D^*$  (an  $JT$ -element column vector), while  $\text{Ln } D_{it}$  is the general element of  $D'$ , an  $N^S T$ -element column vector.)

Since  $\frac{1}{N_{jt}} \sum_{i=1}^{N_{jt}} \text{Ln } D_{it} = \text{Ln } D_{jt}$  (by (3.3)), we use the observed variable  $\text{Ln } D_{jt}$  as the dependent variable on (3.5) to obtain the corresponding coefficient estimates. Finally, we obtain  $\hat{\text{Ln } D_{it}}$  by substituting all the estimated coefficients in (3.5) into equation (3.4), and use  $\text{Ln } D_{it} = \text{Ln } w_{it} - \text{Ln } y_{it}$  to obtain an estimate of  $\text{Ln } y_{it}$ , or  $\text{Ln } y_{it}^*$ , given by

$$\text{Ln } y_{it}^* = \text{Ln } w_{it} - \hat{\text{Ln } D_{it}}. \tag{3.6}$$

#### 2.4 Spillover effects

At firm level, differences in productivity resulting from, say, having an extra worker with a high-school degree can be obtained by using  $\tilde{\gamma}_E$  from equation (2.3), that is:<sup>12</sup>

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<sup>12</sup> We recall that  $E$  is the proportion of workers with a high school degree in firm  $j$ . Thus, all else constant, if a given worker from firm  $j$  attains the high school degree, we have  $\Delta E = \frac{1}{N_{jt}}$ .

$$\tilde{\gamma}_E = \left( \frac{\Delta y_{jt}}{y_{jt}} \right) / \Delta E \Leftrightarrow \frac{\Delta y_{jt}}{y_{jt}} = \frac{\tilde{\gamma}_E}{N_{jt}}. \quad (4.1)$$

At the worker level, and now using equation (3.1), the semi-elasticity of the individual productivity with respect to schooling is given by  $\tilde{\gamma}_E^i = \left( \frac{\Delta y_{it}}{y_{it}} \right) / \Delta E^i$ . Assuming that the change in the schooling level of worker  $i$  only affects his/her own productivity, then the effect on firm productivity is given by  $\Delta y_{jt} = \frac{0 + \dots + 0 + \Delta y_{it} + \dots + 0}{N_{jt}} = \frac{\Delta y_{it}}{N_{jt}}$ . Moreover, as  $\Delta E^i = 1 - 0 = 1$ , we have:

$$\Delta y_{it} = \tilde{\gamma}_E^i * y_{it} \Leftrightarrow \Delta y_{jt} = \frac{\tilde{\gamma}_E^i}{N_{jt}} y_{it} \Leftrightarrow \frac{\Delta y_{jt}}{y_{jt}} = \frac{\tilde{\gamma}_E^i}{N_{jt}} \frac{y_{it}}{y_{jt}}. \quad (4.2)$$

In the absence of human capital spillovers, equation (4.2) is equivalent to equation (4.1). However, we empirically prove that this is not the case, and the difference between (4.1) and (4.2),  $S_{jt}$ , is the magnitude of schooling spillovers, that is:

$$S_{jt} = \left( \tilde{\gamma}_E - \tilde{\gamma}_E^i \frac{y_{it}}{y_{jt}} \right) * \frac{1}{N_{jt}}, \quad (4.3)$$

or, in proportion of the total benefits,

$$s_{jt} = \frac{\left( \tilde{\gamma}_E - \tilde{\gamma}_E^i \frac{y_{it}}{y_{jt}} \right) * \frac{1}{N_{jt}}}{\frac{\tilde{\gamma}_E}{N_{jt}}} = \frac{\left( \tilde{\gamma}_E - \tilde{\gamma}_E^i \frac{y_{it}}{y_{jt}} \right)}{\tilde{\gamma}_E}. \quad (4.4)$$

This modeling can then be easily extended to obtain the spillover effects arising from additional training or a change in the worker skill level, inter al.

### 3. Data

Our linked employer-employee dataset (*LEED*) was obtained by combining *Quadros de Pessoal* (worker-level information) and *Balanço Social* (firm-level information), both from *Gabinete de Estudos e Planeamento* (GEP) of the Portuguese Ministry of Labor. These two

datasets are linked using the unique identification number allocated to each firm. *Quadros de Pessoal* covers the entire population of firms with at least one employee excluding public administration, while firms in *Balanço Social* have at least 100 employees. By construction, all firms in *Balanço Social* are necessarily in the *Quadros de Pessoal* database.

*Balanço Social* includes information on value added, annual worker earnings, the number of employees, hours worked, sector of activity, and region. It also contains information on firm average characteristics of workers (for example age, gender, schooling, tenure, and skill categories). A key feature of *Balanço Social* is that it contains detailed information on firm-provided training, including the number of training sessions (on- and off-the-job), the number and share of training participants by occupation level and the number of training hours.

The information on individual worker attributes is extracted from *Quadros de Pessoal*. It contains monthly earnings, hours of work, age, gender, schooling level, skill, tenure, occupation, and whether the individual is a full or part-time worker, *inter al.*<sup>13</sup> Based on information provided by *Balanço Social*, we have also imputed training participation at worker level. The imputed training variable is then used in worker-level regressions. (The imputation procedures are available from the authors upon request.)

The estimation sample was obtained by applying several filters to the raw *LEED* dataset.<sup>14</sup> We also focused on manufacturing. On the whole we have in our sample 288,129 workers and 1,174 firms. All firms have at least 100 employees and were observed in 1998 and 1999.

[Table 1 near here]

The summary statistics of the estimation sample are presented in Table 1, both at firm and worker level. The first notable finding is that the wage dispersion is very high, both across firms and workers. In column (2), the coefficient of variation is equal to 0.4, which is

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<sup>13</sup> By aggregating worker information at firm level, we can check the corresponding information extracted from *Balanço Social*.

<sup>14</sup> Job switchers, part-time workers, individuals aged less than 16 or more than 65, apprentices, individuals with earnings less than the statutory minimum wage were eliminated from the worker sample, as well as those who were employed in firms located in Madeira and Açores (to establish territorial contiguity).

much higher, for example, than the value observed in Germany, at 0.1 (Addison, Teixeira and Zwick, 2010, Table 1a). Similar heterogeneity is detected in productivity levels across firms. Secondly, the difference between worker- and firm-level (weighted) means, in columns (1) and (2), respectively, is small, a result solely due to the fact that ‘atypical’ workers were dropped from the corresponding sample (see footnote 14). Finally, the standard deviation of earnings, age, schooling, gender and tenure in column (1) are roughly  $\frac{1}{2}$  of the corresponding value in column (2), an indication that there is a sizeable sorting of individuals across firms.

## 4. Results

### 4.1 Productivity and wages at firm level

Columns (1) and (2) of Table 2 present the results from the productivity equations, without and with control for unobserved firm fixed effects, respectively, while columns (3) and (4) give the corresponding estimates of the wage equations. Columns (1) and (3) – and columns (2) and (4) as well – are estimated simultaneously to test for the independence of the error terms. As a matter of fact, the corresponding  $\chi^2$  statistic of the Breusch-Pagan test rejects the null hypothesis of independence of the two models ( $P > \chi^2 = 0.0005$ ). Based on the  $\bar{R}^2$  statistic, the included variables explain approximately 64% of the productivity variability in column (1), increasing slightly to 68% in column (2).

[Table 2 near here]

The coefficient of the capital intensity variable in the productivity equation is similar to the reported values in the literature (e.g. Dearden, Reed and Reenen, 2006, and Hellerstein and Neumark, 1999). We note that in Hellerstein and Neumark (1999) the capital variable in the wage equation is intended to capture firm unobserved effects which are expected to be positively correlated with capital. By comparing columns (3) and (4), there is indeed some evidence in favor of this hypothesis in the sense that the positive and statistically significant effect of capital on firm wages obtained in column (3) virtually vanishes after the introduction of  $\hat{\psi}_j$  in column (4). However, given the difference in parameter estimates between the two

columns – generated by the presence of firm fixed effects – it seems that capital is a poor *proxy* for unobserved firm heterogeneity.

As expected, schooling and training have a positive impact on both productivity and wages, but with the two variables having a bigger effect on the former outcome than on the latter. The introduction of firm effects reduces the schooling and training coefficients, an indication that firm unobserved heterogeneity is positively correlated with human capital variables.

The hypothesis of constant returns to scale (CRS) is not rejected by the data, with  $P > |t| = 0.413$  in case of model (2.3). Under CRS we have therefore  $\tilde{\gamma}_E = \alpha(\gamma_E - 1) = 0.318$ , which implies  $\gamma_E = \frac{0.318}{0.775} + 1 = 1.41$ .<sup>15</sup> In other words, we estimate that workers with at least a high-school degree are 41% more productive than their counterparts with a lower schooling level. Our estimate of the wage gap between these two worker categories is nevertheless much smaller, at 21.7% (column (4), row 1).

Regarding the training variable, and without controlling for firm-specific effects, the semi-elasticity of training with respect to productivity is approximately twice as large as the semi-elasticity of training with respect to wages (at 0.099 and 0.046 in columns (1) and (3), respectively). The gap is even bigger after controlling for unobserved effects, with the corresponding coefficients being equal to 0.060 and 0.004, respectively. This means that, after controlling for unobserved heterogeneity, training has still a clear impact on productivity, with training participants being 7.7% more productive than non-participants;<sup>16</sup> but not on wages as the corresponding coefficient is not statistically different from zero, in column (4).

A quick measure of the percentage of benefits from human capital investment captured by workers can also be easily derived (e.g. Ballot, Fakhfakh and Taymaz, 2006). Thus, and for the training variable, using (2.3) and (2.4) we have  $\left(\frac{dy}{dT}\right)^* \frac{1}{y} = \alpha(\gamma_T - 1)$  and

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<sup>15</sup> From column (2), we have:  $\alpha + \beta - 1 = 0 \Leftrightarrow \alpha = 1 - 0.225 = 0.775$ .

<sup>16</sup> Under constant returns to scale, we have  $\gamma_T = 1.077$ .

$\left(\frac{dw}{dT}\right) * \frac{1}{w} = \lambda_T$ . Then, since  $\frac{dy}{dT} = \alpha(\gamma_T - 1) * y$  and  $\frac{dw}{dT} = \lambda_T * w$ , the worker and firm shares

from training are given by  $\frac{\lambda_T}{\alpha(\gamma_T - 1)} * \frac{w}{y}$  and  $1 - \frac{\lambda_T w/y}{\alpha(\gamma_T - 1)}$ , respectively.

In our sample,  $w/y$  is, on average, equal to 37%. This means that only 2.5% (=  $\frac{\lambda_T}{\gamma_T} * \frac{w}{y} = \frac{0.004}{0.060} * 0.37$ ) of the productivity gains from training are captured by workers.<sup>17</sup> In the case of schooling, the worker share is substantially higher, at 25.2% (=  $\frac{\lambda_E}{\gamma_E} * \frac{w}{y} = \frac{0.217}{0.318} * 0.37$ ). These results – presented in Table 3, column (1) – confirm our priors as skills acquired through schooling are considerably more general than those obtained via workplace training.

[Table 3 near here]

Tenure has also a positive impact on productivity. On average, workers with higher tenure are 14.2% more productive ( $=\alpha(\gamma_s - 1) = 0.11 \Leftrightarrow \gamma_s = \frac{0.11}{0.775} + 1 = 1.142$ ), in column (2) of Table 2. But, as expected, the impact on wages is larger than on productivity, at 19.9% in column (4). In contrast, the variables *top managers and professionals* and *highly skilled and skilled personnel* seem to have a higher impact on productivity than on wages. Interestingly, in both the productivity and wage equations, the evidence points to a negative correlation between tenure and unobserved effects, as the coefficient on tenure actually increases after controlling for unobserved effects.

#### 4.2 Productivity and wages at worker level

As described in section 2.3, (log) worker productivity,  $\ln y_{it}^*$ , can be obtained by using expression (3.6). Then, by aggregating at firm level, we obtain  $\sum_{i=1}^{N_{jt}} \frac{y_{it}^*}{N_{jt}}$ , which can then be compared with the observed firm-level productivity,  $y_{jt}$ . And the result is quite striking:

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<sup>17</sup> We recall that  $\alpha(\gamma_T - 1) = \tilde{\gamma}_T$ .

we find a correlation coefficient of approximately 0.9. Clearly, our measure of worker productivity fits the observed firm data.

We therefore use the obtained estimate of worker productivity to run model (3.1), without and with control for firm and worker fixed effects – in columns (1) and (2) of Table 4, respectively. Columns (3) and (4) give the results from the wage equations. Again the determinants of both productivity and wages are obtained assuming no independence in the error terms. Summarizing the main results, we observe that there is a substantial reduction in all estimated coefficients from columns (1) to (2) and from (3) to (4).<sup>18</sup> The null of absence of unobserved effects is always rejected.

[Table 4 near here]

By comparing columns (2) and (4) we extend the original Hellerstein, Neumark and Troske (1999) investigation to the worker level. The test on the equality of coefficients across equations is easily rejected. For example, in case of schooling we have  $F_{(1, 408.551)} = 547.59$ . The conclusion is that schooling, training, gender, and worker skill level have a higher effect on productivity, while the contribution of tenure to individual wages is higher than to individual productivity. And, interestingly enough, in both equations the impact of firm and worker fixed effects are similar, which means that the corresponding contribution to the productivity and wages is approximately the same at worker level.

### *4.3 Spillover effects*

By comparing the productivity equations at firm and worker level in columns (1) and (2) of Tables 2 and 4, we observe that the corresponding coefficients tend to be smaller at worker level. The fall in the schooling coefficient in Table 4, for example, is particularly pronounced. Following the Dearden, Reed and Reenen (2006) interpretation, this result means that we did find evidence in favor of the existence of schooling spillovers across workers within the same firm. The differences between firm- and worker-level estimates are even higher when we compare columns (2) in Tables 2 and 4. This is an expected result given that

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<sup>18</sup> Again the exception is the tenure variable, which seems to be negatively correlated with the unobserved effects.

in worker level regressions we are able to control for unobserved worker ability, while at the firm level it is only possible to control for the firm average of workers' unobserved effects.<sup>19</sup>

Next, and similarly to Section 4.1, we derived a measure of the relative benefits captured by workers, now based on worker-level estimates. Thus, for each selected covariate, Table 3 shows the workers' share based in two separate regressions, at firm and worker level, in columns (1) and (2), respectively. We found, in particular, that schooling, training, the share of high skilled and skilled workers, and gender imply a higher firm share when models are estimated at firm level, meaning that the gap between productivity and wages is smaller in worker-level than firm-level equations. We have therefore the important result that human capital externalities are indeed larger if they are obtained using the productivity effects, rather than based on wages, with the additional implication that spillovers are in practice mostly captured by firms, as we will show next with more precision.

We use equation (4.4) to explicitly obtain an estimate of human capital spillovers. In the case of schooling, for example, and setting the average worker in the firm as the representative worker  $i$ , the schooling spillover effect in proportion of total benefits,  $s_{E,jt}$ , is given by:

$$s_{E,jt} = \frac{\left( \tilde{\gamma}_E - \tilde{\gamma}_E^i \frac{y_{it}}{y_{jt}} \right)}{\tilde{\gamma}_E} = \frac{(0.318 - 0.036)}{0.318} = 0.887.$$

Similar computations for the training variable and the share of highly skilled and skilled workers yield  $s_{T,jt} = 0.75$  and  $s_{Q_4,jt} = 0.8$ , respectively.

Next, we investigate how the marginal benefit from schooling are shared by: (a) worker  $i$ , through a higher individual wage; (b) worker  $k$ ,  $k \neq i$ , through an increase in firm wage in excess of the individual wage effect; and (c) firm  $j$ , through an increase in firm productivity in excess of the increase in the firm wage.

Using equation (4.1), the total increase in firm productivity is given by  $\Delta y_{jt} = \frac{\tilde{\gamma}_E}{N_{jt}} y_{jt}$ ,

while the worker  $i$ 's wage increase is  $\frac{\lambda_E^i}{N_{jt}} w_{it}$ . In percentage of total benefits, the gains

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<sup>19</sup> We note that our worker-level data exclude job switchers and part-time workers. Re-computing the firm-level variables so that all individuals are included and re-running the models yields similar results.



captured by worker  $i$  are thus given by  $\frac{\lambda_E^i}{N_{jt}} w_{it} / \frac{\tilde{\gamma}_E}{N_{jt}} y_{jt} = \frac{\lambda_E^i}{\tilde{\gamma}_E} \frac{w_{it}}{y_{jt}}$ , or using the coefficient

estimates from Tables 2 and 4,  $\frac{\lambda_E^i}{\tilde{\gamma}_E} \frac{w_{it}}{y_{jt}} = \frac{0.031}{0.318} * 0.37 = 3.5\%$ .

For its part, the change in the firm wage is given by  $\Delta w_{jt} = \frac{\lambda_E}{N_{jt}} w_{jt}$ , which implies that the benefits captured by co-workers in proportion of total benefits are given by

$$\left( \frac{\lambda_E}{N_{jt}} w_{jt} - \frac{\lambda_E^i}{N_{jt}} w_{it} \right) / \frac{\tilde{\gamma}_E}{N_{jt}} y_{jt} = \frac{\left( \lambda_E - \lambda_E^i * \frac{w_{it}}{w_{jt}} \right) w_{jt}}{\tilde{\gamma}_E y_{jt}} = \frac{(0.217 - 0.031)}{0.318} * 0.37 = 21.7\% .$$

We have therefore that 74.8% (=100 - 3.5 - 21.7) of total benefits are captured by firms as the effect on productivity is larger than the increase in the wage of worker  $i$  and the wage of co-workers. Note that according to our computation in Section 4.1, the worker share in productivity gains in the case of the schooling variable is equal to 25.2%. Finally, by comparing the change in worker  $i$ 's productivity with the increase in the wage of worker  $i$ , we obtain the gain in individual productivity that goes to firms, that is:

$$\left( \frac{\tilde{\gamma}_E^i}{N_{jt}} y_{it} - \frac{\lambda_E^i}{N_{jt}} w_{it} \right) / \frac{\tilde{\gamma}_E}{N_{jt}} y_{jt} = \frac{\left( \tilde{\gamma}_E^i - \lambda_E^i * \frac{w_{it}}{y_{jt}} \right) y_{jt}}{\tilde{\gamma}_E y_{jt}} = \frac{(0.036 - 0.031 * 0.37)}{0.318} = 7.7\% .$$

This decomposition is presented in Figure 1, where from left to right we have the increase in wage of worker  $i$  (i.e. 3.5%), the effect on worker productivity net of the wage increase (i.e. 7.7%), and the total spillover effect, which in this illustration amounts to 88.8% (= 67.1 + 21.7). Clearly, the lion's share of spillovers is captured by the firm in a percentage that exceeds 75% of the total.

[Figure 1 near here]

Now, since the median number of workers in firm  $j$  in our dataset is 289, it follows that the benefit per co-worker is approximately 2.15% of the benefit obtained by worker  $i$ , or  $[(0.217/289)/0.035]*100$ . This percentage is very much in line with the literature. For example, Battu, Belfield and Sloane (2003, pp.575-6) report that "an additional year of any single co-worker's education is worth about 3.2% of an additional own year of education." Similarly, in case of Raymond and Roy (2011), the wage increase to worker  $k$  amounts to 2.8% of the additional wage obtained by worker  $i$ .

## 5. Conclusions

We examine in this study the determinants of productivity at firm level based on a highly disaggregated production function in which the labour input is divided into several (observed) worker categories. The micro foundation for the corresponding firm wage equation is based on a standard Mincerian earnings regression with worker and firm fixed effects. Both firm-level productivity (i.e. output per hour) and wage (i.e. earnings per hour) equations assumed to be a function of a common set of regressors. Simultaneous estimation of the two equations using a unique matched employer-employee dataset then reveals that tenure has a greater impact on wages than on productivity, while schooling, training, and skill have a greater impact on productivity. In the case of the productivity gains from training, we show, in particular, that they are almost totally captured by firms.

Secondly, and in a new departure, we estimate wage and productivity equations at worker level. We started by developing a model that allows us to obtain an estimate of the (unobserved) individual productivity, and based on this procedure we show that the generated firm-level productivity is highly correlated with the sample firm average. Not surprisingly, we confirm that schooling, in comparison with firm-provided training, for example, has a greater impact on wages, with the latter implying that 83% of the productivity gains go to firms, while the former implies a worker share of 32%, a result that confirms those obtained from firm-level equations. The introduction of worker and firm unobserved effects into the regression models reduces the schooling and training coefficients, an indication in turn that firm and worker unobserved heterogeneity are, as expected, positively correlated with human capital observed variables.

By comparing the productivity equations at worker and firm level, we found that the corresponding coefficients of the schooling, training, and highly skilled and skilled variables tend to be smaller in worker level estimations than in the corresponding firm-level equations, a result that strongly suggests the existence of spillovers across workers within firms. In this context, we proposed a new model to estimate the magnitude of spillovers, one that is based on comparing results of human capital estimates in productivity equations at worker and firm level. In the case of schooling, for example, we conclude that 89% of the marginal productivity gains are spillovers, with 21.5% being captured by co-workers through a higher wage and the remaining 67.5% by the firm. Putting it differently, our approach shows that the external benefits from an increased worker schooling level are overwhelmingly captured by third parties (i.e. co-workers and firms), a result that certainly shows that social returns to

human capital investments are large *and* greater than private returns. We also have shown that the wage change is actually a poor *proxy* for the real magnitude of human capital spillovers within firms.

## Appendix A

Consider the augmented Mincerian earnings equation at worker level, given by

$$\text{Ln } w_{it} = \text{Ln } A_w + \sum_{r=1}^{10} \lambda_r^i X'_{r,it} + \sum_{p=1}^P \eta_p^w Z_{p,jt} + \beta^w \text{Ln } k_{jt} + \alpha_i + \phi_j + \xi_{it}, \quad (\text{A.1})$$

where,  $\text{Ln } w_{it}$  is the (log) hourly earnings for worker  $i$  in period  $t$ .  $\text{Ln } A_w$  is a constant, while  $Z_{jt}$  denotes the set of characteristics of firm  $j$  in which worker  $i$  is employed in year  $t$ , and  $\text{Ln } k_{jt}$  is the logarithm of capital intensity.  $X'_{it}$  comprises all the dummy variables flagging worker-level characteristics.  $\alpha_i$  denotes the (time-invariant) unobserved ability of worker  $i$ , while  $\phi_j$  is the (time-invariant) unobserved effect specific to firm  $j$ .

Estimation of model (A.1) by *OLS* faces two major obstacles. The first one is related to the possible correlation between observable characteristics and unobserved worker heterogeneity. (And indeed both the standard Hausman and the F-statistic tests comfortably reject the null of no correlation between the unobservable effects and  $X'$ .)<sup>20</sup> The second difficulty is related to proper estimation of parameters  $\alpha_i$  and  $\phi_j$ , given that both the number of workers and firms is very large.

Aggregating over all workers in firm  $j$  in period  $t$  and dividing by  $N_{jt}$  (i.e. the number of workers in firm  $j$  in period  $t$ ), we have:

$$\overline{\text{Ln } w_{jt}} = \text{Ln } A_w + \sum_{r=1}^{10} \lambda_r^i X'_{r,jt} + \sum_{p=1}^P \eta_p^w Z_{p,jt} + \beta^w \text{Ln } k_{jt} + \bar{\alpha}_j + \phi_j + e_{jt}, \quad (\text{A.2})$$

with  $\overline{\text{Ln } w_{jt}} = \frac{1}{N_{jt}} \sum_{i=1}^{N_{jt}} \text{Ln } w_{it}$ .  $\bar{\alpha}_j$  is the worker average unobserved ability in firm  $j$ , with  $\bar{\alpha}_j + \phi_j = \psi_j$ .

Subtracting equation (A.2) from (A.1), we get:

$$\begin{aligned} \text{Ln } w_{it} - \overline{\text{Ln } w_{jt}} = & \\ & \left( \text{Ln } A_w + \sum_{r=1}^{10} \lambda_r^i X'_{r,it} + \sum_{p=1}^P \eta_p^w Z_{p,jt} + \beta^w \text{Ln } k_{jt} + \alpha_i + \phi_j + \xi_{it} \right) - \\ & - \left( \text{Ln } A_w + \sum_{r=1}^{10} \lambda_r^i X'_{r,jt} + \sum_{p=1}^P \eta_p^w Z_{p,jt} + \beta^w \text{Ln } k_{jt} + \bar{\alpha}_j + \phi_j + e_{jt} \right) \Leftrightarrow \end{aligned}$$

<sup>20</sup> The F-statistic is used to test the significance of  $\gamma$  in the auxiliary regression given by:

$$y_{it} - \hat{\lambda} \bar{y}_i = (1 - \hat{\lambda}) \mu + (x_{it} - \hat{\lambda} \bar{x}_i) \beta + (x_{it} - \bar{x}_i) \gamma + e_{it}.$$

$$\Leftrightarrow \text{Ln } w_{it} - \overline{\text{Ln } w_{jt}} = \sum_{r=1}^{10} \lambda_r^i (X'_{r,it} - X_{r,jt}) + (\alpha_i - \bar{\alpha}_j) + (\xi_{it} - e_{jt}), \quad (\text{A.3})$$

with  $\theta_i = \alpha_i - \bar{\alpha}_j$ , which is the (time-invariant) unobserved worker ability in deviation form from the worker average unobserved ability in firm.

It is fair to assume that the difference between the expected individual wage and the expected firm average wage, conditional on worker and firms' characteristics, or  $E(\text{Ln } w_{it} | X'_{it}, K_{jt}, Z_{jt}) - E(\overline{\text{Ln } w_{jt}} | X_{jt}, K_{jt}, Z_{jt})$ , depends on the gap between worker's observed attributes and the mean attributes of his/her counterparts in firm  $j$ ,  $X'_{it} - X_{jt}$ . In this context, it follows that any unexplained wage difference is expected to be due to the difference in unobserved ability from the corresponding firm average. In other words, if the observed ability of a given individual is exactly equal to the observed firm average *and* her/his wage is higher (lower) than the firm average, then her/his unobserved ability must be higher (lower) than that of the fellow co-workers.

In matrix notation, the equation (A.3) is equivalent to

$$W - \bar{W}^i = (X' - X) \lambda^i + C \theta + (\xi - e), \quad (\text{A.4})$$

where  $C$ , we recall, denotes a  $N^s T \times N^s$  matrix of dummies flagging the worker over the sample period  $T$ . Multiplying equation (A.4) by  $M_C = I - P_C$ , where  $P_C$  denotes the matrix that provides an orthogonal projection in  $C$ , we obtain

$$M_C (W - \bar{W}^i) = M_C (X' - X) \lambda^i + M_C C \theta + M_C (\xi - e). \quad (\text{A.5})$$

By definition,  $M_C C \theta = 0$ , and therefore

$$M_C (W - \bar{W}^i) = M_C (X' - X) \lambda^i + M_C (\xi - e), \quad (\text{A.6})$$

which, by Frisch-Waugh's theorem, yields the same estimates and residuals as model (A.4).

The corresponding estimator of  $\lambda^i$  can be then written as

$$\hat{\lambda}^i = \left[ (X' - X)^T M_C (X' - X) \right]^{-1} (X' - X)^T M_C (W - \bar{W}^i). \quad (\text{A.7})$$

From equation (A.4) and (A.7), we finally obtain  $\hat{\theta}$ :

$$\hat{\theta} = (C^T C)^{-1} C^T \left( W - \bar{W}^i - (X' - X) \hat{\lambda}^i \right). \quad (\text{A.8})$$

Let us now consider the matrix of orthogonal projection in  $F$ ,  $P_F = F(F^T F)^{-1} F^T$ , and the matrix  $M_F$ , given by  $M_F = I - P_F$ . Multiplying equation (2.2) by  $M_F$ , we have:

$$M_F \bar{W} = M_F X \lambda + M_F K \beta_w + M_F Z \eta_w + M_F F \psi + M_F e, \quad (\text{A.9})$$

where the second element of the matrix  $M_F Z$ , for example, is given by  $z_{2;1,1} - \frac{z_{2;1,1} + z_{2;1,2}}{2}$ .<sup>21</sup>

By definition, we have  $M_F F \psi = 0$ , which means we can easily compute  $\hat{\lambda}$ ,  $\hat{\beta}_w$ , and  $\hat{\eta}_w$ . Finally, we estimate the unobserved effects of firms by using fixed effects applied to the difference between the observed average wage of the firm and the expected average wage, given the set of covariates:

$$\hat{\psi} = (F^T F)^{-1} F^T \left( \bar{W} - X \hat{\lambda} - K \hat{\beta}_w - Z \hat{\eta}_w \right). \quad (\text{A.10})$$

## Appendix B

Further manipulation of (3.2) yields:

$$\begin{aligned} \text{Ln } w_{jt} - \frac{1}{N_{jt}} \text{Ln} \prod_{i=1}^{N_{jt}} w_{it} &= \text{Ln } y_{jt} - \frac{1}{N_{jt}} \text{Ln} \prod_{i=1}^{N_{jt}} y_{it} \Leftrightarrow \\ \Leftrightarrow \text{Ln} \prod_{i=1}^{N_{jt}} w_{it} - N_{jt} \text{Ln } w_{jt} &= \text{Ln} \prod_{i=1}^{N_{jt}} y_{it} - N_{jt} \text{Ln } y_{jt} \Leftrightarrow \\ \Leftrightarrow \text{Ln} \prod_{i=1}^{N_{jt}} \left( \frac{w_{it}}{w_{jt}^{N_{jt}}} \right) &= \text{Ln} \prod_{i=1}^{N_{jt}} \left( \frac{y_{it}}{y_{jt}^{N_{jt}}} \right) \Leftrightarrow \prod_{i=1}^{N_{jt}} \left( \frac{w_{it}}{w_{jt}^{N_{jt}}} \right) = \prod_{i=1}^{N_{jt}} \left( \frac{y_{it}}{y_{jt}^{N_{jt}}} \right) \Leftrightarrow \\ \Leftrightarrow \frac{\prod_{i=1}^{N_{jt}} w_{it}}{w_{jt}^{N_{jt}}} &= \frac{\prod_{i=1}^{N_{jt}} y_{it}}{y_{jt}^{N_{jt}}}. \end{aligned} \quad (\text{B.1})$$

The following numerical example shows that ultimately what is required in order to condition (B.1) – or (3.2) in the text – be satisfied is that the two coefficients of variation – of worker earnings and worker productivity – are not too further apart. And of course in the process we are not requiring that the two distributions have the same mean and standard deviation which would be unrealistic. Indeed, we expect worker wages to have a smaller

<sup>21</sup>  $z_{2;1,1}$  ( $z_{2;1,2}$ ) denotes the first characteristic of firm 1 in period 1 (2).

standard deviation than worker productivity due the fact that collective bargaining, minimum wages and other labor institutions and regulations play a role in wage developments.

Let us then assume that we have four workers in firm  $j$ , with individual productivities and wages given by  $\{10.5, 10.5, 13.818, 7.182\}$  and  $\{4.2, 2.8, 4.375, 2.625\}$ , respectively. In this case, the two coefficients of variation are different (but not too different) at 0.258 and 0.261, respectively, while the dispersion in productivity is visibly higher than the dispersion in wages, at 2.709 and 0.915, respectively. Clearly, in this case the variability of both variables in relation to the corresponding mean is such that condition (B.1) holds as both the right- and left-hand sides are exactly equal to 0.900.

Note that in our sample – made up of firms with at least 100 employees, we recall – condition (B.1) is easily satisfied, given that both the left- and right-hand sides of the expression converge to zero for a relatively large  $N_j$ . Indeed, in our data, the left-hand side of (B.1) is less than 0.166 in more than 90 percent of the cases. It follows in this context that condition (B.1) seems to be a rather straightforward assumption to make.

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Figure 1: The decomposition of the marginal benefit of schooling

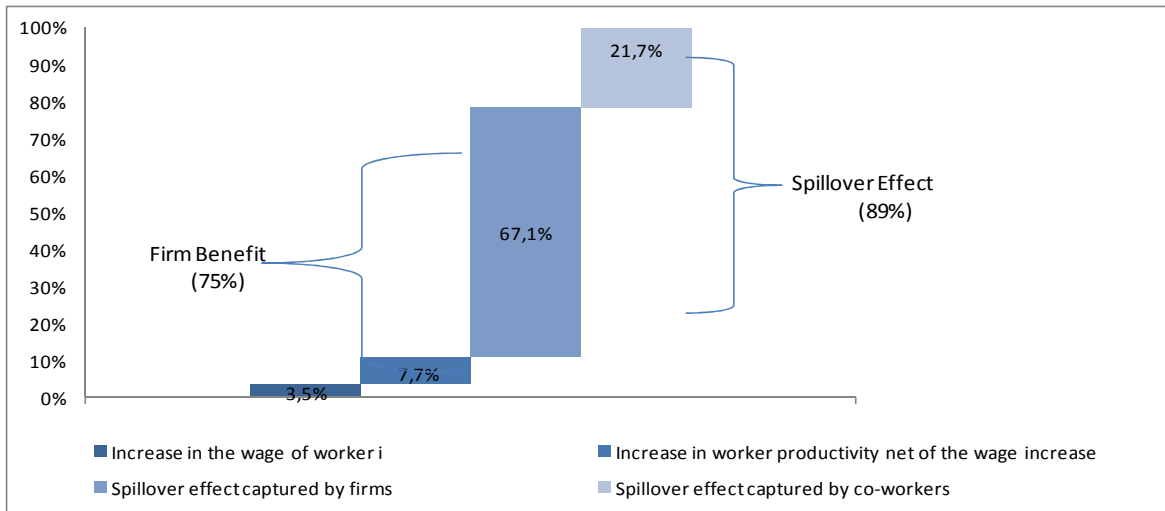


Table 1: Summary statistics at worker and firm level

Variables	Firm Level (1)	Worker Level (2)
(log) Productivity	2.517 (0.794)	2.614 (0.771)
(log) Earnings	1.325 (0.386)	1.406 (0.559)
Schooling	0.192 (0.158)	0.202 (0.401)
Training	0.523 (0.692)	0.413 (0.492)
Tenure	0.466 (0.273)	0.520 (0.499)
Age	0.572 (0.285)	0.603 (0.489)
Gender (male)	0.566 (0.285)	0.624 (0.484)
Top managers and professionals	0.030 (0.036)	0.050 (0.218)
Other managers and professionals	0.048 (0.062)	0.033 (0.178)
Foremen and supervisors	0.055 (0.049)	0.080 (0.272)
Highly skilled and skilled	0.411 (0.256)	0.544 (0.498)
Semiskilled	0.302 (0.268)	0.215 (0.411)
Unskilled	0.099 (0.176)	0.072 (0.259)
Capital	0.696 (1.199)	
Hours per employee	1,770 (2,094)	
Productivity bonus	0.239 (0.498)	
Proportion of full-time workers	0.895 (0.098)	
Proportion of fixed-term	0.077 (0.114)	
Foreign ownership	0.323 (0.468)	
Medium/large firm	0.644 (0.479)	
Norte	0.463	
Centro	0.173	
Lisboa e Vale do Tejo	0.326	
Alentejo	0.024	
Algarve	0.002 (0.044)	
Number of employees	621.9 (1,132)	
Number of observations	1,716	454,346

Standard deviations in parentheses.

Notes: The worker-level information is based on *Quadros de Pessoal*, while the firm-level information is extracted from *Balanço Social*. The balanced panel of manufacturing firms covers 1998 and 1999. Firm-level statistics are weighted by the number of workers in each firm so that columns (1) and (2) are comparable. The (log) productivity at worker level in the first row of the table is obtained using expression (3.6) in the text. The description of variables is presented in Appendix Table 1.

Table 2: Determinants of productivity and wages, firm-level estimates

Variables	Productivity		Wages	
	Without control for unobserved firm effects (1)	With control for unobserved firm effects (2)	Without control for unobserved firm effects (3)	With control for unobserved firm effects (4)
Schooling	0.640 (5.49)	0.318 (2.79)	0.569 (13.39)	0.217 (10.84)
Training	0.099 (4.12)	0.060 (2.60)	0.046 (5.28)	0.004 (0.98)
Tenure	0.092 (1.60)	0.110 (2.01)	0.179 (8.59)	0.199 (19.68)
Age	0.269 (2.53)	0.056 (0.54)	0.100 (2.58)	-0.133 (-7.38)
Gender (male)	0.365 (5.75)	0.260 (4.25)	0.265 (11.43)	0.150 (13.96)
Top managers and professionals	1.583 (3.58)	1.135 (2.68)	0.904 (5.60)	0.414 (5.55)
Other managers and professionals	0.649 (2.39)	0.234 (0.89)	0.808 (8.14)	0.353 (7.69)
Foremen and supervisors	0.249 (1.15)	0.086 (0.42)	0.281 (3.58)	0.103 (2.85)
Highly skilled and skilled	0.146 (2.01)	0.129 (1.85)	0.082 (3.08)	0.063 (5.13)
Semiskilled	0.035 (0.46)	0.072 (0.99)	0.002 (0.06)	0.043 (3.31)
Capital	0.262 (20.80)	0.225 (18.23)	0.032 (7.02)	-0.008 (-3.49)
Productivity bonus	0.006 (0.34)	-0.001 (-0.02)	0.004 (0.63)	-0.003 (-0.96)
Proportion of full-time workers	0.224 (2.00)	0.349 (3.25)	-0.233 (-5.71)	-0.097 (-5.12)
Proportion of fixed-term	-0.073 (-0.71)	-0.020 (-0.20)	-0.037 (-0.99)	0.021 (1.21)
Foreign ownership	0.166 (5.53)	0.112 (3.88)	0.086 (7.88)	0.028 (5.46)
Medium/large firm	0.014 (0.51)	-0.012 (-0.45)	0.033 (3.34)	0.005 (1.14)
Norte	-0.012 (-0.38)	-0.061 (-1.96)	-0.077 (-6.52)	-0.131 (-23.80)
Centro	-0.099 (-2.71)	-0.177 (-5.00)	-0.061 (-4.60)	-0.147 (-23.57)
Alentejo	-0.218 (-2.33)	-0.240 (-2.68)	-0.035 (-1.02)	-0.058 (-3.70)
Algarve	-0.518 (-2.83)	-0.454 (-2.60)	-0.111 (-1.67)	-0.042 (-1.36)
Unobserved firm fixed effect		0.836 (12.64)		1.000 (78.65)
Number of observations	1,701	1,701	1,701	1,637
$F$	76.978	86.215	152.03	787.42
$\bar{r}^2$	0.6438	0.6751	0.7812	0.9506

t-statistics in parentheses.

Notes: Columns (1) and (2) present the estimates from model (2.3), without and with unobserved effects, respectively; columns (3) and (4) reproduce model (2.4), with no control for the unobserved fixed effects in column (3). The description of variables is presented in column (1) of Appendix Table 1. The model also includes a constant, 27 industry dummies, and 2 dummies flagging the legal status of the firm.

Table 3: Worker share from human capital investment

Variables	Firm-level estimates (1)	Worker-level estimates (2)
Schooling	25.2%	31.7%
Training	2.5%	17.2%
Tenure	66.9%	41.1%
Gender	21.4%	32.3%
Top managers and professionals	13.5%	33.4%
Other managers and professionals	55.8%	34.6%
Foremen and supervisors	44.3%	34.5%
Highly skilled or skilled personnel	18.1%	35.8%
Semiskilled	22.1%	34.0%

Note: The corresponding shares are given by  $\frac{\lambda_R}{\alpha(\gamma_R - 1)} * \frac{w}{y}$ , for all  $R$ ,  
 $R = E, T, S, G, Q_1, Q_2, Q_3, Q_4, Q_5$ .

Table 4: Determinants of productivity and wages, worker-level estimates

Variables	(Estimated) Productivity		Wages	
	Without control for unobserved worker and firm effects (1)	With control for unobserved worker and firm effects (2)	Without control for unobserved worker and firm effects (3)	With control for unobserved worker and firm effects (4)
Schooling	0.196 (123.20)	0.036 (54.57)	0.192 (122.19)	0.031 (50.94)
Training	0.119 (91.81)	0.015 (28.33)	0.111 (86.60)	0.007 (14.30)
Tenure	0.102 (91.81)	0.132 (285.74)	0.117 (106.26)	0.146 (345.07)
Age	-0.016 (-15.24)	-0.055 (-123.53)	-0.021 (-20.21)	-0.060 (-147.35)
Gender (male)	0.231 (187.88)	0.095 (184.78)	0.219 (180.47)	0.083 (174.22)
Top managers and professionals	1.039 (329.36)	0.225 (154.40)	1.029 (330.38)	0.203 (151.74)
Other managers and professionals	0.766 (221.96)	0.199 (133.23)	0.760 (223.13)	0.186 (135.50)
Foremen and supervisors	0.502 (191.66)	0.149 (133.10)	0.498 (192.42)	0.139 (135.38)
Highly skilled and skilled	0.213 (106.39)	0.080 (95.91)	0.211 (106.88)	0.077 (100.90)
Semiskilled	0.051 (23.57)	0.025 (27.87)	0.050 (23.08)	0.023 (27.63)
Capital	0.313 (523.71)	0.254 (961.05)	0.068 (115.38)	0.013 (51.59)
Productivity bonus	-0.001 (-0.73)	0.001 (12.17)	-0.001 (-0.86)	0.001 (13.64)
Proportion of full-time workers	0.233 (42.10)	0.262 (114.92)	0.152 (27.88)	0.185 (87.99)
Proportion of fixed-term	-0.001 (-0.28)	-0.013 (-8.25)	0.007 (1.86)	-0.008 (-5.52)
Foreign ownership	0.221 (169.76)	0.110 (197.26)	0.127 (98.50)	0.019 (37.60)
Medium/large firm	-0.025 (-20.28)	-0.043 (-85.80)	-0.009 (-7.40)	-0.028 (-59.24)
Norte	-0.004 (-2.83)	-0.096 (-176.58)	-0.074 (-57.43)	-0.165 (-330.96)
Centro	-0.103 (-59.69)	-0.201 (-281.72)	-0.099 (-58.59)	-0.197 (-299.99)
Alentejo	-0.264 (-59.90)	-0.264 (-144.99)	-0.074 (-16.53)	-0.072 (-42.99)
Algarve	-0.536 (-35.27)	-0.465 (-74.11)	-0.083 (-5.54)	-0.015 (-2.59)
Unobserved worker fixed effect		0.945 (1,188.07)		0.961 (1,314.87)
Unobserved firm fixed effect		1.048 (909.22)		1.015 (957.57)
Number of observations	408,723	408,723	408,723	408,723
$F$	51,453.41	336,757.60	23,270.83	208,624.20
$R^2$	0.8271	0.9706	0.6839	0.9533

t-statistics in parentheses.

Notes: Columns (1) and (2) present the estimates from model (3.1), without and with unobserved effects, respectively; columns (3) and (4) reproduce model (2.1), with no control for the unobserved worker and firm fixed effects in column (3). The description of variables is presented in column (2) of the Appendix Table 1. See notes to Table 2.

Appendix Table 1: Description of the selected variables

Variable	Firm level	Worker level
(log) Productivity ( $Ln y$ )	Log ratio of annual gross value added to hours worked.	It is given by $Ln y_{it}^*$ in model (3.6).
(log) Earnings ( $Ln w$ )	Log of total monthly earnings divided by hours of work.	Log of total monthly earnings divided by hours of work.
Schooling ( $E$ )	Proportion of workers with at least a high-school degree.	Dummy: 1 if the worker has at least a high-school degree; 0 otherwise.
Training ( $T$ )	Proportion of workers who have participated in firm provided training.	Dummy: 1 if the worker has participated in firm provided training; 0 otherwise. This variable has been imputed using a procedure available on request.
Tenure ( $S$ )	Proportion of workers with 10 or more years of service.	Dummy: 1 if the worker has 10 or more years of service; 0 otherwise.
Age ( $O$ )	Proportion of workers between 25 and 44 years old.	Dummy: 1 if the worker has more than 25 years old and less than 44 years old; 0 otherwise.
Gender (male) ( $G$ )	Proportion of male workers.	Dummy: 1 if the worker is male; 0 otherwise.
Top managers and professionals ( $Q_1$ )	Proportion of top managers and professionals.	Dummy: 1 if the worker is top manager or professional; 0 otherwise.
Other managers and professionals ( $Q_2$ )	Proportion of other managers and professionals.	Dummy: 1 if the worker is other manager or professional; 0 otherwise.
Foremen and supervisors ( $Q_3$ )	Proportion of foremen and supervisors.	Dummy: 1 if the worker is foreman or supervisor; 0 otherwise.
Highly skilled and skilled ( $Q_4$ )	Proportion of highly skilled and skilled personnel.	Dummy: 1 if the worker is highly skilled or skilled; 0 otherwise.
Semiskilled ( $Q_5$ )	Proportion of semiskilled personnel.	Dummy: 1 if the worker is semiskilled; 0 otherwise.
Unskilled ( $Q_6$ )	Proportion of unskilled personnel.	Dummy: 1 if the worker is unskilled; 0 otherwise.
(log) Capital ( $Log k$ )	(Log) Capital stock per hour of work. The stock of capital is proxied by the annual volume of capital depreciation.	
Productivity bonus	Ratio of non-standard compensation to <i>basic</i> earnings.	
Proportion of full-time workers	Proportion of full-time workers.	
Proportion of fixed-term	Proportion of fixed-term contract workers.	
Foreign ownership	Dummy: 1 if the firm is owned partial or totally by foreigners; 0 otherwise.	
Medium/large firm	Dummy: 1 if the number of employees is more than 250; 0 otherwise.	
Norte/Centro/Lisboa e Vale do Tejo/Alentejo/Algarve	Dummy: 1 if the firm is located in Norte/Centro/Lisboa e Vale do Tejo/Alentejo/Algarve; 0 otherwise.	
Unobserved firm fixed effects	It is given by $\hat{\psi}_j$ in model (B.10).	
Unobserved worker fixed effects	Corresponds to $\hat{\theta}_i$ in model (B.8).	

Note: The variables at firm (worker) level are extracted from *Balanço Social (Quadros de Pessoal)*.